113. Method of Least Work.—The so-called theorem or principle of "Least Work" is closely related to the subject of elastic deflections just considered in its availability for furnishing equations of condition in addition to those of a purely statical character in cases where indetermination would result without them. This principle of least work is expressed in the simple statement that when any structure supports external loading the work performed in producing elastic deformation of all the members will be the least possible. Although this principle may not be susceptible of a complete and general demonstration, it may be shown to hold true in many cases if not all. The hypothesis is most reasonable and furnishes elegant solutions in many useful problems.

The application of this principle requires the determination of expressions for the work performed in the elastic lengthening and shortening of pieces subjected either to tension or compression, and for the work performed in the elastic bending of beams carrying loads at right angles to their axes. Both of these expressions can be very simply found.

Let it be supposed that a piece of material whose length is L and the area of whose cross-section is A is either stretched or compressed by the weight or load S applied so as to increase gradually from zero to its full value. The elastic change of length will be $\frac{SL}{AE}$, E being the coefficient of elasticity. The average force acting will be $\frac{1}{2}S$, hence the work performed in producing the strain will be

$$\frac{1}{2}\frac{S^2L}{AE}. \qquad (48)$$

It will generally be best, although not necessary, to take L in inches. The expression (48) applies either to tension or compression precisely as it stands.

To obtain the expression for the work performed by the stresses in a beam bent by loads acting at right angles to its axis, a differential length (dL) of the beam is considered at any normal section in which the bending moment is M, the total length being L. Let I be the moment of inertia of the normal section, A, about an axis passing through the centre of gravity of the latter, and let k be the intensity of stress (usually the stress per square inch) at any point distant d from the axis about which I is taken. The elastic change produced in the indefinitely short length dL when the intensity k exists is $\frac{k}{E}dL$. If dA is an indefinitely small portion of the normal section, the average force or stress, either of tension or compression, acting through the small elastic change of length just given, can be written by the aid of equation (5) as

$$\frac{1}{2}k\,.\,dA = \frac{Md}{2I}\,.\,dA. \qquad (49)$$

Hence the work performed in any normal section of the member, for which M remains unchanged, will be, since $\int k\,.\,dA\,.\,d = M$,

$$\int \frac{M}{2IE}kd\,.\,dA\,.\,dL = \frac{M^2}{2EI}dL. \qquad (50)$$

The work performed throughout the entire piece will then be

$$\int \frac{M^2}{2EI}dL. \qquad (51)$$

Each of the expressions (48) and (51) belongs to a single piece or member of the structure. The total work performed in all the pieces subjected either to direct stress or to bending, and which, according to the principle of least work, must be a minimum, is found by taking the summation of the two preceding expressions:

$$e = \frac{1}{2E}\sum \frac{S^2L}{A} + \frac{1}{2E}\sum \int \frac{M^2}{I}dL = \text{minimum}. \qquad (52)$$

In making an application of equation (52) it is to be remembered that S is the direct stress of tension or compression in any member, and that M is the general value of the bending moment in any bent member expressed in terms of the length L.

114. Application of Method of Least Work to General Problem.—The problem which generally presents itself in the use of equation (52) is the finding of an equation which expresses the condition that the work expended in producing elastic deformation shall be a minimum, some particular stress in the structure or some external load or force being the variable. If t represent that variable, then the desired equation of condition will be found simply by placing the first differential coefficient of e in equation (52) equal to zero:

$$\frac{de}{dt}=\frac{1}{E}\left(\sum\frac{S}{A}\frac{dS}{dt}dL+\sum\int\frac{M}{I}\frac{dM}{dt}dL\right)=0. \quad . \quad (53)$$

The solution of equation (53) will give a value of t which will make the work performed as expressed in equation (52) a minimum. This method is not a difficult one to employ in such cases as those of drawbridges and stiffened suspension bridges. In the latter case particularly it is of great practical value.

115. Application of Method of Least Work to Trussed Beam.—The method of least work may be illustrated by the application of the preceding equations to the simple truss shown in Fig. 32. The pieces BC and GD are supposed to be of yellow-pine timber, the former 10 inches by 14 inches (vertical) in section and the latter 8 inches by 10 inches, while each of the pieces BD and DC are two $1\frac{5}{8}$-inch round steel bars. The coefficient of elasticity E will be taken at 1,000,000 pounds for the timber and 28,000,000 for the steel. The length of BC is 360 inches; GD 96 inches; $BD=96\times2.13=204.5$ inches.

$$\tan\alpha=1.875 \quad \text{and} \quad \sec\alpha=2.13.$$

The weight W resting at G is 20,000 pounds. A part of this weight is carried by BC as a simple timber beam, while the remainder of the load will be carried on the triangular frame BCD acting as a truss, the elastic deflection of the latter throwing a part of the load on BC acting as a beam. According to the principle of least work the division of the load will be such as to make the work performed in straining the different members of the system a minimum.

That part of W which rests on BC as a simple beam may be represented by W_1, while W_2 represents the remaining portion carried by the triangular frame. As G is at the centre of the span, the beam reaction at either B or C is $\frac{1}{2}W_1$. Hence the

Hence

$$\frac{de}{dW_2} = .000,007,46W_2 - .000,426(W - W_2) = 0. \quad . \quad . \quad (56)$$

The solution of this equation gives

$$W_2 = .893W = 19,660 \text{ pounds.}$$
$$W_1 = \quad 340 \quad \text{``}$$

It is interesting to observe that the first term of the second member of equation (56) is the deflection of the point of application of W_2 as a point in the frame, while the second term is the deflection of the point of application of W_1 considered as a point of the beam. In other words, the condition resulting from the application of the principle of least work is equivalent to making the elastic deflections by W_1 and W_2 equal. Indeed equation (53) expresses the equivalence of deflections whenever the features of the problem are such as to involve concurrent deflections of two different parts of the structure.

116. Removal of Indetermination by Methods of Least Work and Deflection.—The indetermination existing in connection with the computations for such trusses as those shown in Fig. 22 and Fig. 23 can be removed by finding equations of condition by the aid of the method of least work or of deflections. It is evident that the component systems of bracing of which such trusses are composed must all deflect equally. Hence expressions may be found for the deflections of those component trusses, each under its own load. Since these deflections must be equal, equations of condition at once result. A sufficient number of such equations, taken with those required by statical equilibrium, can be found to solve completely the problem. Such methods, however, are laborious, and the ordinary assumption of each system carrying wholly the loads resting at its panel-points is sufficiently near for all ordinary purposes.

The method of least work can be very conveniently used for the solution of a great number of simple problems, like that which requires the determination of the four reactions under the four legs of a table, carrying a single weight or a number of weights, and many others of the same character.

CHAPTER X.

117. The Arched Rib, of both Steel and Masonry.—During the past ten or fifteen years the type of bridge structure called the arched rib has come into much use, and its merits insure for it a wider application in the future. It partakes somewhat of the nature of both truss and arch; or it may be considered a curved beam or girder. The ordinary beam or truss when placed in a horizontal position and loaded vertically yields only vertical reactions. Under the same conditions, however, the arched rib will produce both vertical and horizontal reactions, and the latter must either be resisted by abutments of sufficient mass, or by a tie-rod, usually horizontal, connecting the springing points of the rib.

The arched rib may be built solid, as was done in the early days of bridge-building in this country when engineers like Palmer, Burr, and Wernwag introduced timber arches in combination with their wooden trusses, or as a curved plate girder, one of the most prominent examples of which is the Washington Bridge across the Harlem River in the city of New York; or, again, as a braced frame or curved truss, like the 800 feet arched rib carrying the roadway traffic and trolley cars across the Niagara gorge, or like those used in such great railroad train-sheds as the Grand Central Station, New York, the Pennsylvania stations at Jersey City and Philadelphia, and the Philadelphia and Reading station in Philadelphia. Those are all admirable examples of steel arched ribs, and they are built to sustain not only vertical loads but, in the case of station roofs, the normal or horizontal wind pressures.

Within a few years, less than ten, another type of arched rib has been brought into use and promises to be one of the most beautiful as well as the most substantial applications of this type of structure; that is, the arched rib of combined steel and concrete. Many examples of this type of structure already exist both in this country and in Europe, probably the most prominent of which in this country is that at Topeka, Kansas, across the Kansas River.

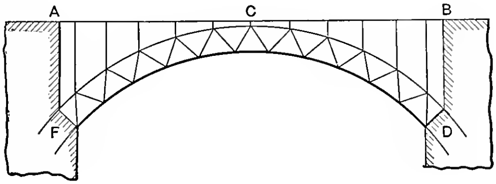


FIG. 33.

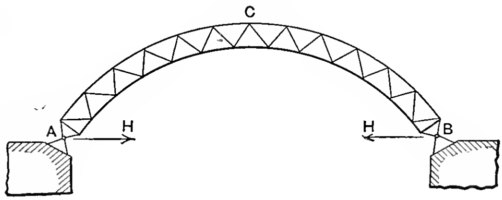


FIG. 34.

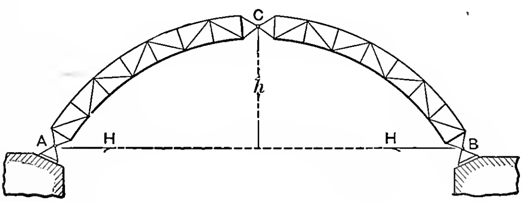


FIG. 35.

The characteristic feature of this type of structure, so far as the stresses developed in it are concerned, is the thrust throughout its length, more or less nearly parallel to its axis, which is combined with the bending moments and shears similar to those found in ordinary bridge-trusses. This thrust is the arch characteristic and differentiates it in a measure from the ordinary bridge-truss, while the bending moments and shears to which it is subjected differentiate it, on the other hand, from the pure arch type or a series of blocks in which thrust only exists. The thrust, bending moments, and shears in arched ribs are all affected by certain principal features of design. Those features are either fixedness of the ends of the ribs or the presence of pin-joints at those ends or at the crown. Fig. 33 represents an arched rib with its ends D and F supposed to be rigidly fixed in masonry or by other effective means.

118. Arched Rib with Ends Fixed.—The railroad steel arched bridge at St. Louis, built by Captain Eads between 1868 and 1874, is a structure of this character. The three spans (two each 537 feet 3 inches and one 552 feet 6 inches in length from centre to centre of piers) consist of ribs the main members of which are composed of chrome steel. It was a structure of unprecedented span when it was built, and constituted one of the boldest pieces of engineering in its day. The chords of the ribs are tubes made of steel staves, and their ends are rigidly anchored to the masonry piers on which they rest. It is exceedingly difficult, indeed impossible, to fix rigidly the ends of such a structure, and observations in this particular instance have shown that the extremities of the ribs are not truly fixed, for the piers themselves yield a little, giving elastic motion under some conditions of loading.

119. Arched Rib with Ends Jointed.—The rib shown in Fig. 34 is different from the preceding in that pin-joints are supplied at each end, so that the rib may experience elastic distortion or strain by small rotations about the pins at A and B. In the computations for such a design it is assumed that the ends of the rib may freely change their inclination at those points. As a matter of fact the friction is so great, even if no corrosion exists, as to prevent motion, but the presence of the pins makes no bending moment possible at the end joints, and the failure to move freely probably produces no serious effect upon the stresses in the ribs. The presence of these pin-joints simplifies the computations of stresses and renders them better defined, so that there is less doubt as to the actual condition of stress under a given load than in the type shown in Fig. 33 with ends fixed more or less stiffly. In Fig. 34, if the horizontal force H exerted by the ends of the rib against the points of support is known, the remaining stresses in the structure can readily be computed;

but neither in Fig. 34 nor in Fig. 33 are statical equations sufficient for the determination of stresses. Equations of condition, depending upon the elastic properties of the material, are required before solutions of the problems arising can be made.

120. Arched Rib with Crown and Ends Jointed.—The rib shown in Fig. 35 possesses one characteristic radically different from any found in the ribs of Figs. 33 and 34, in that it is three-jointed, one pin-joint being at the crown and one at each end. So far as the conditions of stress are concerned, this is the simplest rib of all. Since there is a pin-joint at the crown as well as at the ends, the bending moments must be zero at each of those three points whatever may be the condition of loading. The point of application of the force or thrust at the crown, therefore, is always known, as well as the points of application at the ends of the joints. As will presently be seen, this condition makes equations of statical equilibrium sufficient for the determination of all stresses in the rib, and no equations depending upon the elastic properties of the material are required. The stresses in this class of ribs, therefore, are more easily determined than in the other two, and they are better defined. These qualities have insured for it a somewhat more popular position than either of the other two classes. The ribs of the great train-sheds of the Pennsylvania and Reading railroads in Jersey City and in Philadelphia belong to this class, while those of the Grand Central Station at New York City belong to the class shown in Fig. 34, as does the arched rib across the Niagara gorge, to which reference has already been made.

121. Relative Stiffness of Arch Ribs.—Obviously the three-hinged ribs are less stiff than the two-hinged ribs or those with fixed ends. This is a matter of less consequence for station roofs than for structures carrying railroad loads. The joints of the two-hinged rib being at the ends of the structure, there is but little difference in stiffness between that class of ribs and those with ends fixed. Indeed the difference is so slight, and the uncertainty as to the degree of fixedness of the fixed ends of the rib is so great, that the latter type of rib possesses no real advantage over that with hinged ends.

122. General Conditions of Analysis of Arched Ribs.—In each of the three types of arched ribs shown in Figs. 33, 34, and 35 it is supposed that all external forces act in the vertical planes which contain the centre lines of the various members of the rib. There are, therefore, the three conditions of statical equilibrium expressed by the three equations (35), (36), and (37). In practically all cases, except those of arched ribs employed in roof construction, all the external loads are vertical. In such cases the equations of statical equilibrium of the entire structure may be reduced to two only, viz., equations (36) and (37). These features of the problems connected with the design of arched ribs will always make necessary, except in the case of the three-hinged rib (Fig. 35), equations of condition depending upon the elastic properties of the structure.

The rib represented by Fig. 33 is supposed to have its ends so fixed that the inclinations of the centre line at F and D will never change whatever may be the loading or the variation of temperature. This requires the application at each of those points of a couple whose moment varies in value, but which is always equal and opposite to the bending moment at the same point produced by the loads imposed on the rib. It is also to be observed that the loads resting upon the rib are not divided between the points of support F and D in accordance with the law of the lever, since the conditions of fixedness at the ends are equivalent to continuity. There are then to be found, as acting external to the rib, the two vertical reactions and the two moments at F and D, as well as the horizontal thrust exerted at the ends of the structure, which is sometimes resisted by the tie-rod, making five unknown quantities. Inasmuch as all external loading is supposed to be vertical, equations (36) and (37) are the only statical equations available, and three others, depending upon the elastic properties of the structure, must be supplied in order to obtain the total of five equations of condition to determine the five unknown quantities. Inasmuch as the end inclinations remain unchanged, the total extension or compression of the material at any given constant distance from the axis of the rib taken between the two end sections F and D must be equal to zero. Similarly, whatever may be the amount or condition of loading, the vertical and horizontal deflections of either of the ends F or D in relation to the other must be zero, since no relative motion between these two points can take place. It is not necessary in these lectures to give the demonstration of the equations which express the three preceding elastic conditions, but if M is the general value of the bending moment for any point of the rib, and if x and y are the horizontal and vertical coordinates of the centre line of the rib, taking the central point of the section at either F or D as an origin, those equations, taken in the order in which the elastic conditions have been named, will be the following, in which n represents a short length of rib within which the bending moment M is supposed to remain unchanged.

$$\sum_D^F nM = 0; \quad \sum_D^F nMx = 0; \quad \sum_D^F nMy = 0. \quad . \quad (57)$$

The second and third of these equations express the condition that the vertical and horizontal deflections respectively of the two ends in reference to each other shall be zero. The conditions expressed by equation (57) are constantly used in engineering practice to determine the bending moments and stresses which exist in the arched rib with fixed ends. The graphical method is ordinarily used for that purpose, as its employment is a comparatively simple procedure for a rib whose curvature is any whatever.

If the rib has hinged joints at the ends, as in Fig. 34, obviously there can be no bending moment at either of those two points, and hence the two equations of condition which were required in connection with Fig. 33 to determine them will not be needed. There is, therefore, no restriction as to the angle of inclination of the centre line of the rib at those two points. Again, it is obvious that either end A or B may have vertical movement, i.e., deflection in reference to the other, without affecting the condition of stress in any member of the rib; but it is equally obvious that neither A nor B can be moved horizontally, i.e., deflected in reference to the other, without producing bending in the rib and developing stresses in the various members. The unknown quantities in this case are, therefore, only the horizontal thrust H exerted at the two springing points A and B, and the two vertical reactions, making a total of three unknown quantities, equations for two of which will be given by equations (36) and (37). The other equation required is the third expression in equation (57), expressing the condition that the horizontal deflection of either of the points A or B in respect to the other is zero, since the span AB is supposed to remain unchanged. By the application of the graphical method to this case, as to the preceding, the employment of equations (36), (37), and (58) will afford an easy and quick determination of the three unknown quantities, whatever may be the curvature of the rib.

$$\sum_B^A n\,My=0. \qquad (58)$$

If the reactions and horizontal thrust H are found, stresses in every member may readily be computed and the complete design made.

If the arch is three-hinged, as in Fig. 35, the condition that the bending moment must be zero at the crown C under all conditions of loading gives a third statical equation independent of the elastic properties of the structure which, in connection with equations (36) and (37), give three equations of condition sufficient to determine the two vertical reactions and the horizontal thrust H. In this case, as has already been stated, no elastic equations of condition are required.

The determination of the end reactions, bending moments, and horizontal thrust H, in these various cases, is all that is necessary in order to compute with ease and immediately the stresses in every member of the rib. These computations are obviously the final numerical work required for the complete design of the structure. These procedures are always followed, and in precisely the manner indicated, in the design of arched ribs by civil engineers, whether the rib be articulated, i.e., with open bracing, or with a solid plate web, like those of the Washington Bridge across the Harlem River.

CHAPTER XI.

123. Beams of Combined Steel and Concrete.*—A reference has already been made to a class of beams and arches recently come into use and now quite widely employed, composed of steel and concrete, the former being completely surrounded by and imbedded in the latter. These composite beams are very extensively used in the floors of fire-proof buildings as well as for other purposes. Arches of combined concrete and steel were probably first built in Germany and but a comparatively few years ago. During the past ten years they have been largely introduced into this country, and many such structures have not only been designed but built. The most prominent design of arches of combined concrete and steel are those of the proposed memorial bridge across the Potomac River at Washington, for which a first prize was awarded as the result of a national competition in the early part of 1900. So far as the bending or flexure of these composite beams and arches is concerned, the theory is identically the same for both, the formulæ for each of which are given below. In order to express these formulæ the following notation will be needed:

P is the thrust along the arch determined by the methods explained in the consideration of arched ribs.

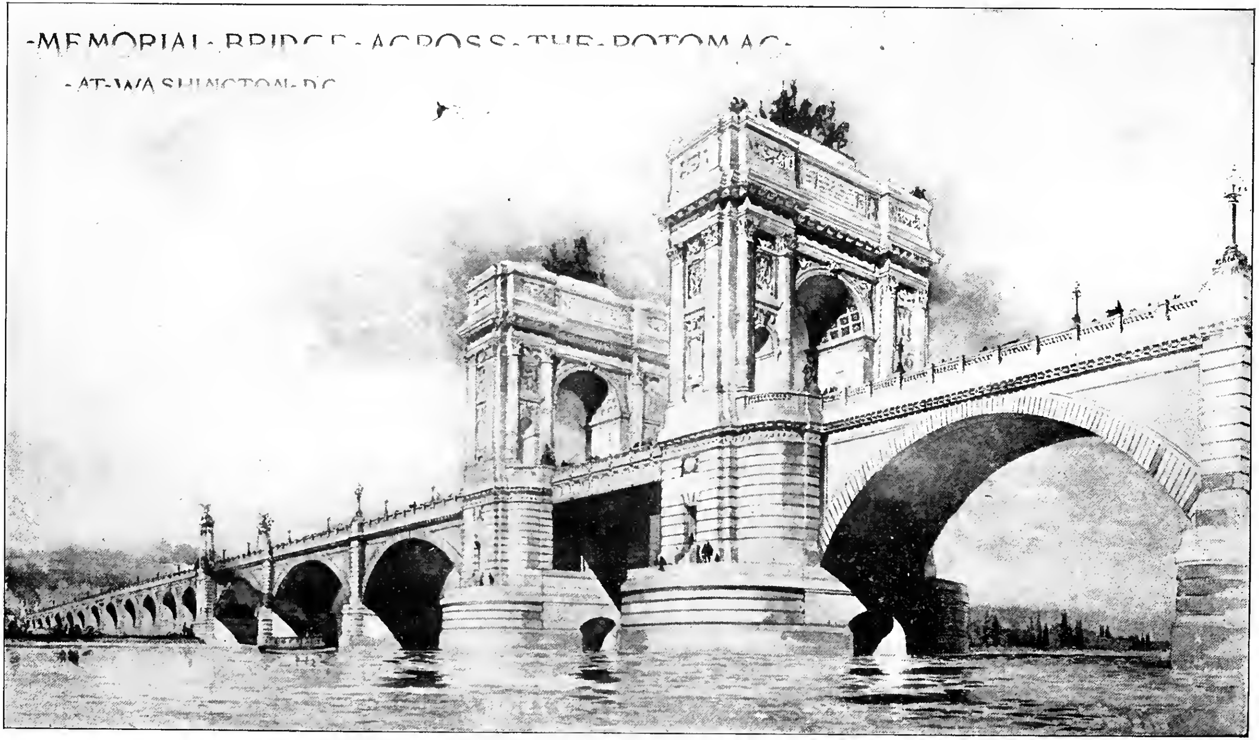
l is the distance of the line of the thrust P from the axis of the arched rib.

E_1 and E_2 are coefficients of elasticity for the two materials.

A_1 and A_2 are areas of normal section of the two materials.

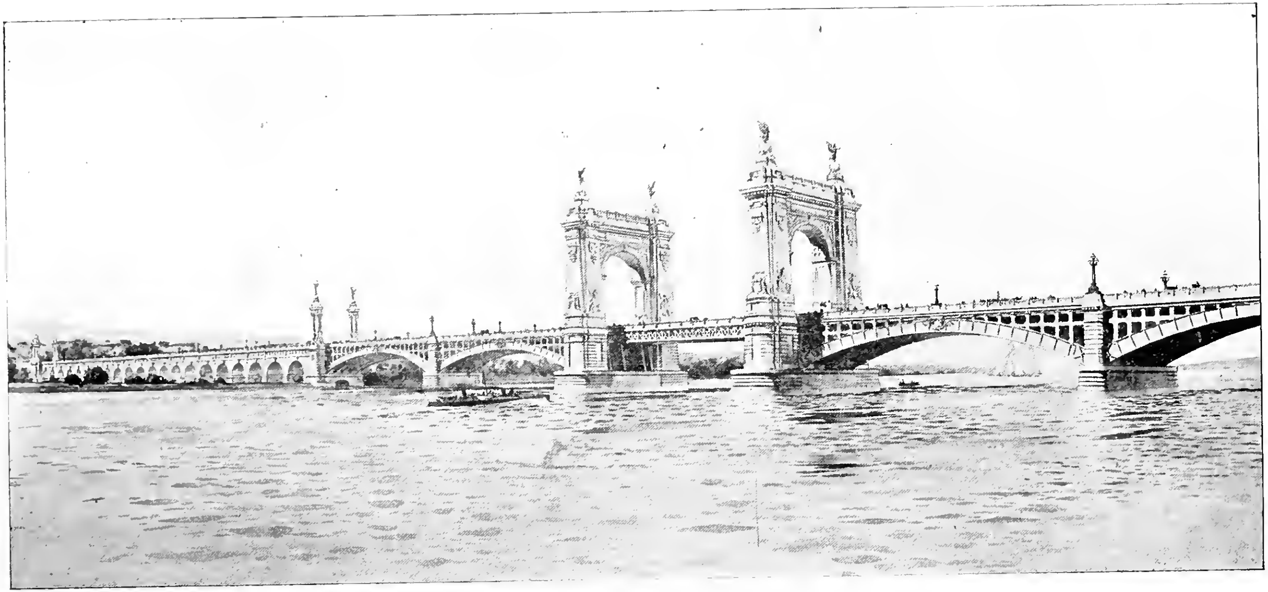
I_1 and I_2 are moments of inertia of A_1 and A_2 about the neutral axes of the composite beam or arch sections.

* For a complete and detailed statement of this whole subject, including design work, reference should be made to the author's "Elasticity and Resistance of Materials."



Wm. H. Burr, Civil Engineer.
E. P. Casey, Associated Architect.

PLAN NO. 2.
Plan Awarded First Prize in National Competition.
River spans 192 feet clear. Total length of structure 3615 feet.



Wm. H. Burr, Civil Engineer.
E. P. Casey, Associated Architect.

PLAN NO. 1.

The Towers of this Plan were Recommended by Board of Award to be Substituted for Those in Plan No. 2.

River spans 283 feet clear. Total length of structure 3437 feet.

k_1 and k_2 are intensities of bending stress in the extreme fibres of the two materials.

h_1 and h_2 are total depths of the two materials.

d_1 and d_2 are distances from the neutral axes to farthest fibres of the two materials; distances to other extreme fibres would be $(h_1 - d_1)$ and $(h_2 - d_2)$.

W_1 and W_2 are loads, either distributed or concentrated, carried by the two portions.

$W = W_1 + W_2$ is total load on the beam or arch.

$$q_1 = \frac{W_1}{W} \quad \text{and} \quad q_2 = \frac{W_2}{W}; \quad \therefore q_1 + q_2 = 1; \quad e = \frac{E_2}{E_1}.$$

The application of the theory of flexure to the case of a beam or arch of two different materials, steel and concrete in this case, will give the following results:

$$M = Pl; \text{ hence } M_1 = q_1 Pl \quad \text{and} \quad M_2 = q_2 Pl. \quad . \quad . \quad (59)$$

$$q_1 = \frac{W_1}{W} = \frac{E_1 I_1}{E_1 I_1 + E_2 I_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

$$q_2 = \frac{W_2}{W} = \frac{E_2 I_2}{E_1 I_1 + E_2 I_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

$$k_1 = \left(\frac{P}{A_1 + eA_2} + \frac{Md}{I_1 + eI_2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

$$k_2 = e\left(\frac{P}{A_1 + eA_2} + \frac{Md}{I_1 + eI_2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

These formulæ exhibit some of the main features of the analysis which must be used in designing either beams or arches of combined steel and concrete. In the use of these equations care must be taken to give the proper sign to the bending moment M. They obviously apply to the combination of any two materials, although at the present time the only two used in such composite structures are steel and concrete. If the subscript 1 belongs to the concrete portion, and the subscript 2 to the steel portion, there may be taken $E_1 = 1,500,000$ to $3,000,000$ and $E_2 = 30,000,000$. Hence $e = 20$ to 10.

The purpose of introducing the steel into the concrete is to make available in the composite structure the high tensile resistance of that metal. A very small steel cross-section is sufficient to satisfactorily accomplish that purpose. The percentage of the total composite section represented by the steel will vary somewhat with the dimensions of the structure and the mode of using the material; it will usually range from 0.75 per cent to 1.5 per cent of the total section. The large mass of concrete in which the steel should be completely imbedded serves not only to afford a large portion of the compressive resistance required in both arches and beams, but also to preserve the steel effectively from corrosion. Many experiments have shown that it requires but a small per cent of steel section to give great tensile resistance to the composite mass.

CHAPTER XII.

124. **The Masonry Arch.**—The masonry arch is so old that its origin is lost in antiquity, but its complete theory has been developed with that of other bridge structures only within the latest period. It is only possible here to give some of the main features of that theory and a few of the fundamental ideas on which it is based. It is customary among engineers to regard the masonry arch as an assemblage of blocks finely cut to accurate dimensions, so that the assumption of either a uniform or uniformly varying pressure in the surface of contact between any two may be at least sufficiently near the truth for all practical purposes. Although care is taken to make joints between ring-stones or voussoirs completely cemented or filled with a rich cement mortar, it is usually the implicit assumption that such joints do not resist tension. As a matter of fact many arch joints are capable of resisting considerable tension, but, in consequence of settlement or shrinkage, cracks in them that may be almost or quite imperceptible frequently prevent complete continuity. It is, therefore, considered judicious to determine the stability of the ordinary masonry arch on the assumption that the joints do not resist tension.

In these observations it is not intended to convey the impression that no analysts treat the ordinary arch as a continuous elastic masonry mass, like the composite arches of steel and concrete. Although much may be said in favor of such treatment for all arches, it is believed that prolonged experience with arch structures makes it advisable to neglect any small capacity of resistance to tension which an ordinary cut-stone masonry joint may possess, in the interests of reasonable security.

The ring-stones or voussoirs of an arch are usually cut to form circular or elliptic curves, or to lines which do not differ sensibly from those curves. The arch-ring may make a complete semicircle, as in the old Roman arches, or a segment of a semicircle; or the stones may be arranged to make a pointed arch, like the Gothic; or, again, a complete semiellipse may be formed, or possibly a segment of that curve. When a complete semiellipse or complete semicircle is formed, the arches are said to be full-centred, and in those cases they spring from a horizontal joint at each end. On the other hand, segmental arches spring from inclined joints at each end called skew-backs.

125. Old and New Theories of the Arch.—In the older theories of the arch, considered as a series of blocks simply abutting against each other, the resultant loading on each block was assumed to be vertical. In the modern theories, on the other hand, the resultant loading on any block is taken precisely as it is, either vertical or inclined, as the case may be. Many arches are loaded with earth over their arch-rings. This earth loading produces a horizontal pressure against each of the stones, as well as a vertical loading due to its own weight. In such cases it is necessary to recognize this horizontal or lateral pressure of the earth, as it is called, as a part of the arch loading.

It is known from the theory of earth pressure that the amount of that pressure per square foot or any other square unit may vary between rather wide limits, the upper of which is called the abutting power of earth, and the latter the conjugate pressure due to its own weight only. If w is the weight per cubic unit of earth and x the depth considered, and if φ be the angle of repose of the earth, the abutting power per square unit will have the value:

$$p = wx\frac{1+\sin\varphi}{1-\sin\varphi}, \quad \ldots \ldots \quad (64)$$

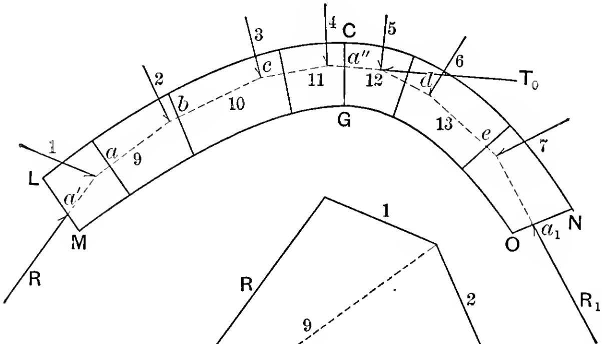
while the horizontal or conjugate pressure due to the weight of earth only will be:

$$p' = wx\frac{1-\sin\varphi}{1+\sin\varphi}. \quad \ldots \ldots \quad (65)$$

The use of these formulæ will be illustrated by actual arch computations.

Fig. 36 is supposed to show a set of ring-stones for an arch of any curvature whatever. The joints LM and ON represent the skew-backs or springing joints, while R and R_1 represent the supporting forces or reactions with centres of action at a' and a_1.

Fig. 36.



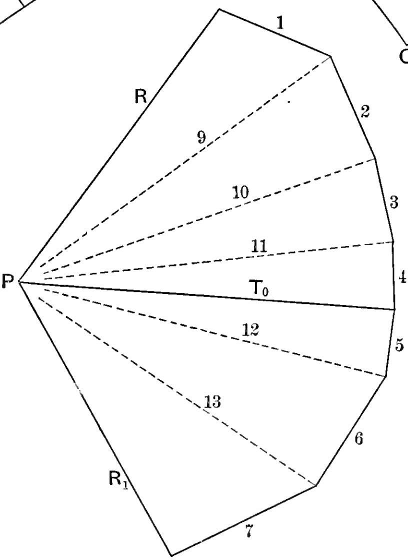


Fig. 37.

The ring is divided into blocks or pieces by the joints at a, b, c, d, and e, the resultant loading or force on each block being given by the lines with arrow-heads and numbered 1, 2, 3, 4, 5, 6, and 7. Fig. 37 represents a force polygon constructed in the ordinary manner by laying off carefully to scale the two reactions R and R_1, together with the loads or forces numbered 1 to 7, inclusive. By constructing the so-called polygonal frame in the ring-stones of Fig. 36 in the usual manner with its lines or sides parallel to the radiating lines in Fig. 37, as shown by the broken lines, the points a, b, c, etc., are found where the resultant forces cut each joint. The line drawn through those points thus determined is called the line of resistance of the arch. Obviously, if that line of resistance be determined, the complete stability or instability of the arch, as the case may be, will be established. Furthermore, the complete determination of the force polygon in Fig. 37, and the corresponding polygonal frame drawn in the arch-ring, constitute all the computations involved in the design of an arch.

The thrust T_0 at the crown, shown both in Fig. 36 and Fig. 37, is frequently horizontal, although not necessarily so ; its value is shown by Fig. 37. In the older arch theories a principle was enunciated called the "principle of least resistance." The thrust T_0 is a fundamental and so-called passive force. That is, its magnitude depends not only upon its position, but also largely upon the magnitude of the active forces which represent the loading on the arch-ring. Under the principle of least resistance it was laid down as a fundamental proposition, in making arch computations, that this passive force T_0 must be the least possible consistent with the stability of the structure. While this provisional proposition answered its purpose well enough, there are other clearer methods of procedure which are thoroughly rational and involve the employment of no extraneous considerations other than those attached to the determination of statical equilibrium.

A scrutiny of the conditions existing in Fig. 36 will show that if the external forces or loadings on the individual blocks of the ring are given, four quantities are to be determined, viz., the two reactions R and R_1 and their lines of action. Inasmuch as no elastic features of the structure are to be considered, there are available for the determination of these four quantities the three equations of equilibrium, equations (35), (36), and (37), which are not sufficient for the purpose. If one line of action, such as that of R, be located by assuming its point of application a', the three equations just named will be sufficient for the determination of the remaining three equations; and that is precisely the method employed. It is tentative, but perfectly practicable. If, instead of assuming one of the points of application of the reactions, we assume both of those points and construct a trial polygonal frame, it will be necessary to use but two of the three equations of statical equilibrium. For that purpose there are employed equations (35) and (36), but in a graphical manner, which will presently be illustrated.

126. Stress Conditions in the Arch-ring.—Before proceeding to the construction of an actual line of resistance, a little consideration must be given to the stress conditions in the arch-ring. As the joints are considered capable of resisting no tension, the dimensions of the arch-ring must be finally so proportioned that pressure only will exist in each and every joint. If each centre of pressure, as a, b, etc., in Fig. 36, is found in the middle third of the joint, it is known from a very simple demonstration in mechanics that no tension will ever exist in that joint, although the pressure may be zero at one extremity and a maximum at the other. This is the condition usually imposed in designing an arch-ring to carry given dead or live loads. It is usually specified that "the line of resistance of the ring must lie in the middle third." It must be borne in mind, however, that the stability of the ring is perfectly consistent with the location of the line of resistance outside of the limits of the middle third, provided it is not so far outside as to induce crushing of the ring-stones. Whenever that crushing begins the arch is in serious danger and complete failure is likely to result.

127. Applications to an Actual Arch.—These principles will be applied to the arch-ring shown in Fig. 38, in which the clear span TU is 90 feet. The radius CO of the soffit (as the under surface of the arch is called) is 50 feet, the ring being circular and segmental. The uniform thickness of the ring shown at the various joints is assumed at 4 feet as a trial value. The loading above the ring to the level of the line $E'O$ is assumed to be dry earth weighing, when well rammed in place, 100 pounds per cubic foot. The depth of this earth filling at the crown n of the arch is taken at 4 feet. The ring-stones are assumed to be of granite or best quality of limestone, weighing 160 pounds per cubic foot. The thickness or width of arch-ring of one foot is assumed, as each foot in width is like every other foot, and the loads are taken for that width of ring. The rectangle $EJJ'E'$

is supposed to represent a moving load covering one half of the span and averaging 500 pounds per linear foot; in other words, averaging 500 pounds per square foot of upper surface projected in the line $E'O$. The total length of the arch-ring, measured on the soffit, is about 113 feet, and it is divided into ten equal portions for the purpose of convenient computation. The radial joints so located are as shown at *de*, *fg*, *hk*. From the points where these joints cut the extrados (as the upper surface of the arch-ring is called) vertical broken lines are erected, as shown in Fig. 38.

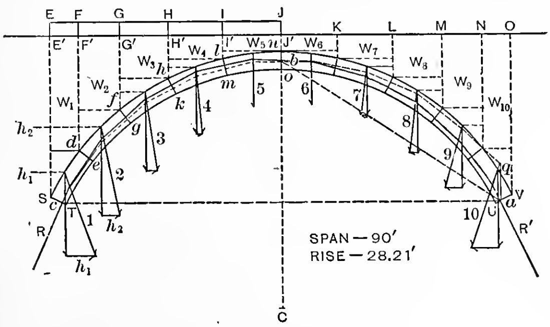


FIG. 38.

The horizontal line drawn to the left from f gives the vertical projection of that part of the extrados between d and f, and the horizontal earth pressure on df will be precisely the same in amount as that on the vertical projection of df, as just found. In the same manner the horizontal earth pressure on that part of the extrados between any two adjacent joints may be found. The mid-depths of these vertical projections below the line $E'O$ are to be carefully measured by scale and then used for the values of x in equations (64) and (65), which now become equations (66) and (67), as the angle of repose φ is taken to correspond to a slope of earth surface of 1 vertical on $1\frac{1}{2}$ horizontal.

$$p = 3.51wx. \qquad (66)$$

$$p' = 0.285wx. \qquad (67)$$

The horizontal earth pressures thus found are as follows:

$$h_1 = \begin{cases} 101{,}500 \text{ pounds;} \\ 8{,}700 \quad \text{"} \end{cases} \qquad h_3 = \begin{cases} 30{,}625 \text{ pounds;} \\ 2{,}625 \quad \text{"} \end{cases}$$

$$h_2 = \begin{cases} 59{,}500 \quad \text{"} \\ 5{,}100 \quad \text{"} \end{cases} \qquad h_4 = \begin{cases} 9{,}800 \quad \text{"} \\ 840 \quad \text{"} \end{cases}$$

These quantities h_1, etc., are found by multiplying the two intensities p and p' by the vertical projections of the surface on which they act. The larger values are found by equation (66) and represent the abutting power of the earth, while the smaller values are found by equation (67), and represent the horizontal or conjugate pressure of the earth due to its own weight only. The actual horizontal earth pressure against the arch-ring may lie anywhere between these limits.

The weights of the moving load, earth, and ring-stones between each pair of vertical lines and radial joints shown in Fig. 38 are next to be determined, and they are as follows:

$W_1 = 27,300$	pounds;	$W_6 = 12,300$	pounds;
$W_2 = 27,900$	"	$W_7 = 15,550$	"
$W_3 = 24,500$	"	$W_8 = 19,500$	"
$W_4 = 21,300$	"	$W_9 = 19,400$	"
$W_5 = 18,300$	"	$W_{10} = 24,300$	"

The centres of gravity of these various vertical forces are shown in Fig. 38 at the points W_1, W_2, etc. The triangles of forces shown in that figure and composed, each one, of a vertical and horizontal force as described, are laid down in actual position on the arch-ring, as shown. All data are thus secured for completing the force polygon and polygonal frame or line of resistance. It will be assumed that the reactions R and R' cut the springing joints at c and a, respectively, one third of the width of the joint from the soffit, and it will further be assumed that b, the mid-point of the joint at the crown, is also in the line of resistance. The assumption of the location of these three points is made for the reason, as is well known, that with a given system of forces a polygonal frame may be found which will pass through any three points in the ring.

The force polygon B, 1, 2, 3, . . . , 10, A, Fig. 39, is then drawn with the loadings on each ring segment found as already explained. The horizontal forces are taken as represented by the smaller values of h_1, h_2, h_3, h_4. Other force polygons with larger values of these horizontal forces were tried and not found satisfactory. Having constructed the force polygon and assumed the trial pole P', the radial lines are drawn from it as shown in

Fig. 39. The polygonal frame shown in broken lines in Fig. 38 results from this trial pole. The frame practically passes through b and c, but leaves the ring, passing outside of it, above the joint VU. The point q in this frame is vertically above a. The "three-point" method of finding the frame that will pass through

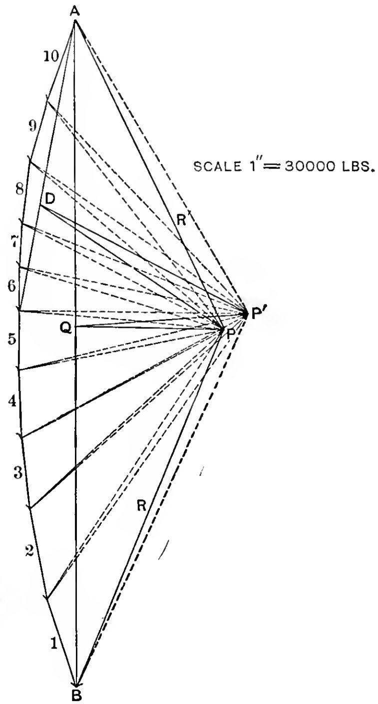


FIG. 39.

a, b, and c was then employed. The line $A6$, Fig. 39, was drawn; then $P'D$ was drawn parallel to qb, Fig. 38 (not shown); after which PD was drawn parallel to ab, until it intercepted the horizontal line PQ, the line $P'Q$ having previously been drawn parallel to qc (not shown). The final pole P was thus found. The polygonal frame shown in full lines in the arch-ring was then drawn with sides parallel to the lines radiating from P, all in accordance with the usual methods for such graphic analysis. That polygonal frame lies within the middle third of the arch-ring, although at three points it touches the limit of the middle third. The arch, therefore, is stable.

This construction shows that, with the actual loading of the ring, a line of resistance can be found lying within the middle third; its stability under the conditions assumed is, therefore, demonstrated. It does not follow that the line of resistance as determined must necessarily exist, since there may be others located still more favorably for stability. This indetermination results from the fact already observed that the equations of statical equilibrium are not sufficient in number to determine the four unknown quantities (the two horizontal and the two vertical reactions); but the process of demonstrating the stability of the arch-ring is simple and sufficient for all ordinary purposes. The line of resistance found, if not the true one, is so near to it that no sensible waste of material is involved in employing it. This indetermination has prompted some engineers and other analysts to consider all arch-rings as elastic, thus obtaining other equations of condition. While such a procedure may be permissible, it is scarcely necessary, and perhaps not advisable, in view of the fact that many joints of cut-stone arches become slightly open by very small cracks, resulting possibly from unequal settlement, quite harmless in themselves, having practically no effect upon the stability of the structure.

128. Intensities of Pressure in the Arch-ring. — It still remains to ascertain whether the actual pressures of masonry in the arch-ring are too high or not. The greatest single force shown in the force polygon in Fig. 39 is the reaction R, having a value by scale of 122,000 pounds, under the left end of the arch, and it is supposed to act at the limit of the middle third of the joint. Hence the average pressure on that joint will be

$$\frac{122,000 \times 2}{4} = 61,000 \text{ pounds per square foot.}$$

This value may be taken as satisfactory for granite or the best quality of limestone.

Again, it is necessary in bridges, as in some other structures, to determine whether there is any liability of stones to slip on each other. In order that motion shall take place the resultant forces acting on the surface of a stone joint must have an inclination to that surface less than a value which is not well determined and which depends upon the condition of the surface of the stone; it certainly must be less than 70°. The inclination of every resultant force in Fig. 38 to the surface on which it acts is considerably greater than that value and, hence, the stability of friction is certainly secured.

129. Permissible Working Pressures.—The working values of pressures permissible on cut-stone and brick or other masonry must be inferred from the results of the actual tests of such classes of masonry in connection with the results of experience with structures in which the actual pressures existing are known. It is safe to state that with such classes of material as are used in the best grade of engineering structures these pressures will generally be found not to exceed the following limits:

Concrete, 20,000 to 40,000 pounds per square foot.

Cement rubble, same values.

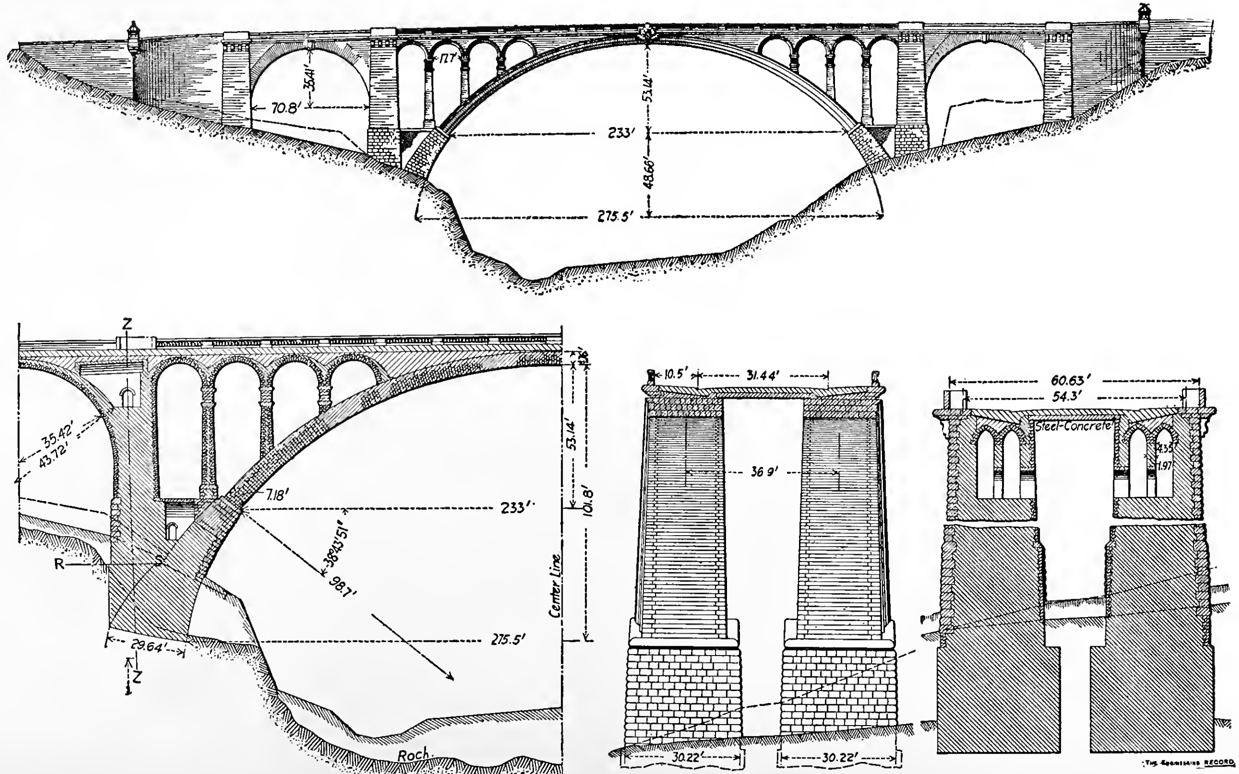
Hard-burned brick, cement-mortar joints, 30,000 to 50,000 pounds per square foot.

Limestone ashlar, 40,000 to 60,000 pounds per square foot.

Granite ashlar, 50,000 to 70,000 pounds per square foot.

The masonry arch is at the same time the most graceful and the most substantial and durable of all bridge structures, and it is deservedly coming to be more and more used in modern bridge practice. One of the greatest railroad corporations in the United States has, for a number of years, been substituting, wherever practicable, masonry arches for the iron and steel structures replaced. The high degree of excellence already developed in this country in the manufacture of the best grades of hydraulic cement at reasonable prices, and the abundance of cut stone, has brought this type of structure within the limits of a sound economy where cost but a few years ago would have excluded it. It is obviously limited in use to spans that are not very great but yet considerably longer than any hitherto constructed.

130. Largest Arch Spans.—The longest arch span yet built has been but recently completed in Germany at the city of Luxemburg. This bridge has a span of 275.5 feet and a rise of 101.8 feet. It is rather peculiarly built in two parallel parts

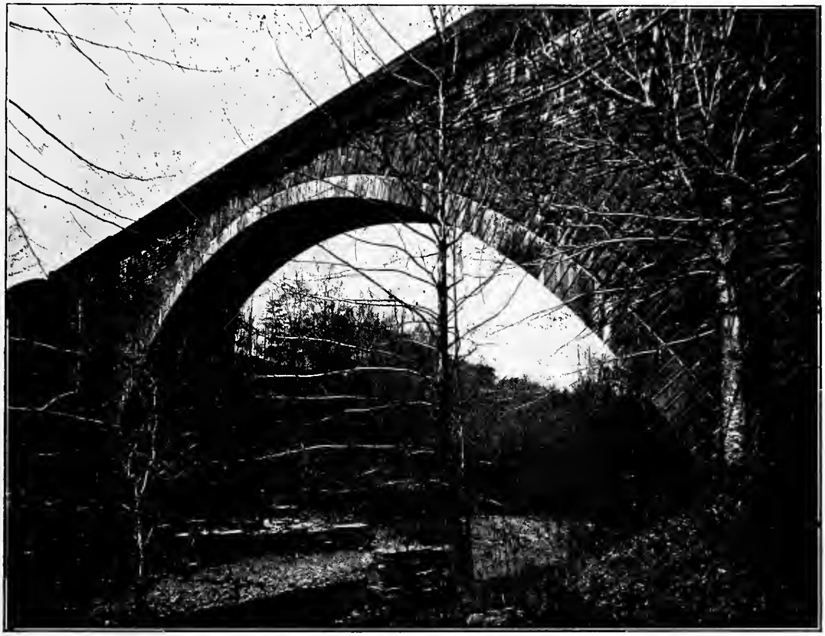


Half Longitudinal Section. Section through Crown. Section through Main Pier at Z-Z.

FIG. 40.—Elevation of Luxemburg Bridge and Sections of Main Span.

separated 19.5 feet in the clear, the space between being spanned by slabs or beams of combined concrete and steel. The arch-ring is 4.75 feet thick at the crown and 7.18 feet thick at a point 53.14 feet vertically below the crown where it joins the spandrel masonry. The roadway is about 52.5 feet wide and 144.5 feet above the water in the Petrusse River, which it spans.

The longest arch in this country is known as the Cabin John Bridge of 220 feet span and 57.5 feet rise. It is a segmental arch and is located a short distance from the city of Washington,



Cabin John Bridge, near Washington, D. C.

carrying the aqueduct for the water-supply of that city. These lengths of span may be exceeded in good ordinary masonry construction, but the high degree of strength and comparative lightness which characterize the combination of steel and concrete will enable bridges to be built in considerably greater spans than any yet contemplated in cut-stone masonry.

CHAPTER XIII.

131. **Cantilever and Stiffened Suspension Bridges.**—There are two other types of bridges of later development which have, in recent years, become prominent by remarkable examples of both completed structure and design; they are known as the cantilever and stiffened suspension bridges. Both are adapted to long spans, although the latter may be applied to much longer spans than the former. A cantilever structure, with a main span of 1800 feet between centres of piers, is now in process of construction across the St. Lawrence River at Quebec, while the well-known Forth Bridge across the Firth of Forth in Scotland has a main span of 1710 feet. The longest stiffened suspension bridge yet constructed is the New York and Brooklyn Bridge, with a river span of about 1595.5 feet between centres of towers, but the stiffened suspension system has been shown by actual design to be applicable to spans of more than 3200 feet, with material now commercially produced.

132. **Cantilever Bridges.**—Figs. 41 and 42 exhibit in skeleton outline two prominent cantilever designs for structures in this

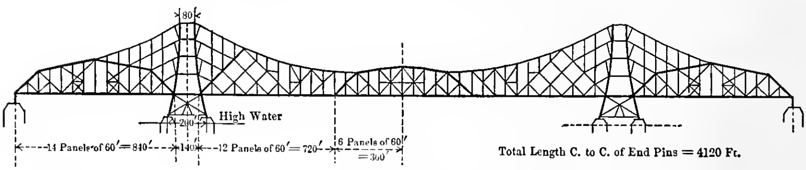


FIG. 41.

country. That shown in Fig. 41 was intended for a bridge across the Hudson River between Sixtieth and Seventieth streets, New York City. The main central opening has a span of 1800 feet, and a length of 2000 feet between centres of towers. Fig. 42 shows the Monongahela River cantilever bridge,* now being

* This bridge was designed by and is being constructed under the direction of Messrs. Boller and Hodge, Consulting Engineers, New York City.

built at Pittsburgh, Penn. Both figures exhibit the prominent features of the cantilever system. The main parts are the towers, at each end of the centre span, which are 534.5 feet high in the North River Bridge and 135 feet high in the Monongahela River structure, and the central main or river span with its simple non-continuous truss hung from the ends of the cantilever brackets or arms which flank it on both sides. These cantilever arms are simply projecting trusses continuous with the shore- or anchor-arms. They rest on the piers at either end of the main span, as a lever rests on its fulcrum. This arrangement requires the shore extremities or the anchor-arms to be anchored down by a heavy weight formed by the masonry piers at those points. Recapitulating and starting from the two shore ends of the structure, there are the anchor-spans, continuous at the towers, with the cantilever arms projecting outward toward the centre of the main opening and supporting at their ends the suspended truss, which is a simple, non-continuous one. It is thus evident that the cantilever bridge is a structure composed of continuous trusses with points of contraflexure permanently fixed at the ends of the suspended span. The greatest bending moments are at the towers, and the great depth at that point is given for the purpose of affording adequate

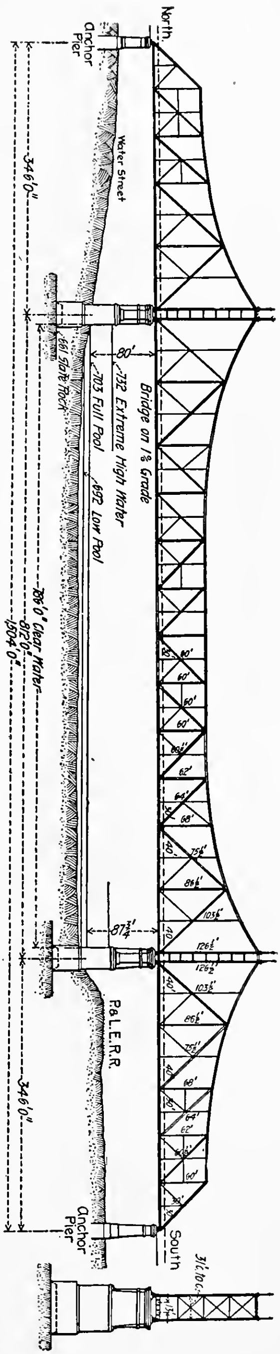


FIG. 42.—Monongahela Bridge—Pittsburgh, Carnegie & Western Railroad (Wabash), at Pittsburgh.

resistance to those moments by the members of the structure. The following statement shows some elements of the more prominent cantilever bridges of this country and of the Forth Bridge:

Name.	Length of Cantilever Opening, Centre to Centre of Towers.	Total Length.
Pittsburgh..............	812 feet.	1504 feet.
Red Rock (Colo.)......	660 "	990 "
Memphis (Tenn.).......	790.48 "	2378.2 "
Forth.................	1710 "	5330 "

The arrangement of web members of cantilever structures is designed to be such as will transfer the loads from the points of application to the points of support in the shortest and most direct paths. Both Figs. 41 and 42 show these general results accomplished by an advantageous arrangement of web members.

It is interesting to note that the first cantilever bridge designed and built in this country was constructed in 1871. This structure was designed and erected by the late C. Shaler Smith, a prominent civil engineer of his day.

133. Stiffened Suspension Bridges.—The stiffened suspension bridge is a structure radically different in its main features and its mode of transferring load to points of support from any heretofore considered, except arched ribs. When a load is supported by a beam or truss, the stresses, either in the web members of the truss or in the solid web of the beams, travel up and down those members in zigzag directions with a relatively large amount of metal required for that kind of transference. That metal is represented by the weight of the web members of the truss and of the solid web of the beam. Again, there are two sets of truss members—the chords or flanges, one of which sustains tension and the other an equal amount of compression. The greater part of this metal must be so placed and used that the working intensities of stress are comparatively small. This is particularly the case in compression members of both chords and webs which constitute the greater portion of the weight of the truss. All compression members are known as long columns which sustain not only direct compression but bending, and the amount of stress or load which they carry per square inch is relatively small, decreasing as the length increases. For all these reasons the amount of metal required for both beams and trusses is comparatively large. In suspension bridges, however, the conditions requiring the employment of a relatively large amount of metal with relatively small unit stresses are absent. The main members of a suspension bridge are the cables and the stiffening trusses, the latter being light in reference to the length of span. The cables are subjected to tension only, which is the most economical of all methods of using metal. A member in tension tends to straighten itself, so that it is never subjected to bending by the load which it carries. The opposite condition exists with compression members. Again, grades of steel possessing the highest ultimate resistance may be used in the manufacture of cables. It is well known that wire is the strongest form in which either wrought iron or steel can be manufactured. While the ultimate tensile resistance of ordinary structural steel will seldom rise above 70,000 pounds per square inch, steel wire, suitable to be used in suspension-bridge cables, may be depended upon, at the present time, to give an ultimate resistance of at least 180,000 pounds per square inch. The elastic limit of ordinary structural steel is but little above half its ultimate resistance, while the elastic limit of the steel used in suspension-bridge cables is probably not less than three fourths of its ultimate resistance. It is seen, therefore, that the high resistance of steel wire makes the steel cable of the suspension bridge a remarkably economical application of metal to structural purposes.

The latest example of stiffened suspension-bridge is the new East River Bridge reaching across the East River from Broadway in Brooklyn to Delancey Street, New York City, now being built, with a main span of 1600 feet between centres of towers. The entire length of the metal structure is 7200 feet, and the elevation of the centres of cable at the tops of the towers is 333 feet above mean high water.

Fig. 43 shows a view of this bridge. Its three principal divisions are the cables, the stiffening trusses, and the towers. The latter afford suitable points of support for the cables, which not only extend over the river span, but are carried back to points on the land where they are securely attached to a heavy mass of anchorage masonry. These anchorages must be sufficiently heavy to prevent any load which may come upon the bridge from moving them by the pull of the cables. It is usual to

Fig. 43.—New East River Bridge.

make these masses so great that they are capable of resisting from two to two and a half times the pull of the cables.

134. The Stiffening Truss.—The function of the stiffening trusses is peculiar and imperatively essential to the proper action of the whole system. If they are absent and a weight should be placed upon the cable at any point, a deep sag at that point would result. If a moving load should attempt to pass along a roadway supported by a cable only, the latter would be greatly distorted, and it would be impossible to use such a structure for ordinary traffic. Some means must then be employed by which the cable shall maintain essentially the same shape and position, whatever may be the amount of loading. It can be readily shown that if any perfectly flexible suspension-bridge cable carries a load of uniform intensity over the span from one tower to the other, the curve of the cable will be a parabola, with its vertex at the lowest point. Furthermore, it can also be shown that if any portion of the span be subjected to a uniform load, the corresponding portion of the cable will also assume a parabolic curve. It is assumed in all ordinary suspension-bridge design that the total weight of the structure, including the cables and the suspension-rods which connect the stiffening trusses to the cable, is uniformly distributed over the span, and that assumption is essentially correct. So far as the weight of the structure is concerned, therefore, the curve of the cable will always be parabolic. It only remains, therefore, to devise such stiffening trusses as will cause any moving load passing on or over the bridge to be carried uniformly to the cables throughout the entire span. This condition means that if any moving load whatever covers any portion of the span, the corresponding pull of the suspension-rods on the cables must be uniform from one tower to the other, and that result can be practically accomplished by the proper design of stiffening trusses; it is the complete function of those trusses to perform just that duty.

135. Location and Arrangement of Stiffening Trusses.—It has been, and is at the present time to a considerable extent, an open question as to the best location and arrangement of the stiffening trusses. The more common method in structures built is that illustrated by the New York and Brooklyn and the new East River bridges. Those stiffening trusses are uniform in depth, extending from one tower to the other, or into the land spans, and connected with the cables by suspension-rods running from the latter down to the lower chords of the trusses. It is obvious that the floor along which the moving load is carried must have considerable transverse stiffness, and hence it may appear advisable to place the stiffening trusses so that the floor may be carried by them. On the other hand, some civil engineers maintain that it is a better distribution of stiffening metal to place it where the cables themselves may form members of the stiffening trusses, with a view to greater economy of material.

Figs. 44, 45, and 46 illustrate some of the principal proposed methods of constructing stiffening trusses in direct connection with the cables. The structure shown in Fig. 44 illustrates the skeleton design of the Point Bridge at Pittsburgh. The curved member is a parabolic cable composed of eye-bars. This parabolic cable carries the entire weight of the structure and moving load when uniformly distributed. If a single weight rests at the centre, the two straight members of the upper chord may be assumed to carry it. If a single weight rests at any other point of the span, it will be distributed by the bracing between the straight and curved members of the stiffening truss. Obviously the most unbalanced loading will occur when one half of the span is covered with moving load. In that case the bow-string stiffening truss in either half of Fig. 44 will make the re-

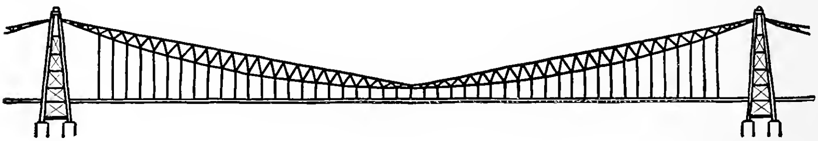


FIG. 44.

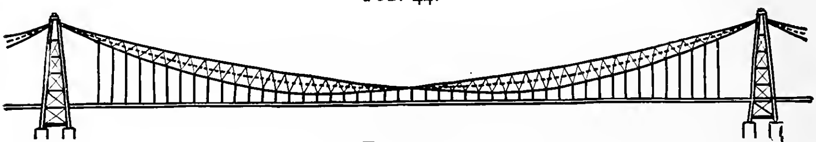


FIG. 45.

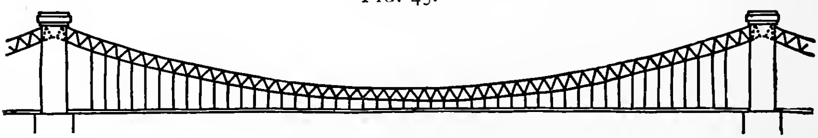


FIG. 46.

quired distribution and prevent the parabolic tension member from changing its form.

The type of bracing shown in Fig. 45 possesses some advantages of a peculiar nature. Each curved lower chord of the stiffening truss corresponds to the position of the perfectly flexible cable with the moving load covering that half of the span which belongs to the greatest sag of the cable. The two parabolic cables thus cross each other in a symmetrical manner at the centre of the span. If the moving load covers the entire span, the line of resistance or centre line of imaginary cable will be the parabola, shown by the broken line midway along each crescent stiffening truss. The diagonal bracing placed between the cables is so distributed and applied as to maintain the positions of cables under all conditions of loading.

The mode of constructing the stiffening truss between two cables, shown in Fig. 46, is that adopted by Mr. G. Lindenthal in his design for a proposed stiffened suspension bridge across the Hudson River with a span of about 3000 feet. The two cables are parabolic in curvature and may be either concentric or parallel. This system of stiffening bracing possesses some advantages of uniformity and is well placed to secure efficient results. The same system has been used in suspension bridges of short span by Mr. Lindenthal at both St. Louis and Pittsburgh. The stiffening bracing produces practically a continuous stiffening truss from one tower to the other, whereas the systems shown in Figs. 44 and 45 involve practically a joint at the centre of the span.

In all these three types of vertical stiffness the floor is designed to meet only the exigencies of local loading, being connected with the stiffening truss above by suspension bars or rods, preferably of stiff section.

When stiffening trusses are placed along the line of the floor, as in the case of the two East River bridges, to which reference has already been made, those trusses need not necessarily be of uniform depth, and they may be continuous from tower to tower or jointed at the centre, like those of the New York and Brooklyn suspension bridge. This centre joint detracts a little from the stiffness of the structure, but in a proper design this is not serious.

136. Division of Load between Cables and Stiffening Truss.—In a case where continuous stiffening trusses are employed it is obvious that they may carry some portion of the moving load as ordinary trusses. The portion so carried will be that which is required to make the deflection of the stiffening truss equal to that of the cable added to the stretch of the suspension-rods. In the old theory of the stiffening truss constructed along the floor of the bridge this effect was ignored, and the computations for the stresses in those trusses were made by the aid of equations of statical equilibrium only. That assumption, that the cable carried the entire load, was necessary to remove the ambiguity which would otherwise exist. In modern suspension-bridge design those trusses may be assumed continuous from tower to tower with their ends anchored at the towers, or they may be designed to be carried continuously through portions of the land spans and held at their extremities by struts reaching down to anchorages, so that those ends may never rise nor fall, but move horizontally if required. If there are no pin-joints in the trusses at the centre and ends of the main span, equations of statical equilibrium are not sufficient to enable the reactions under the trusses and the horizontal component of cable tension to be found.

One of the best methods of procedure for such cases is that of least work, in which the horizontal component of cable tension is so found that the total work performed in the elastic deflection of the stiffening trusses, suspension-rods, cables, and towers is a minimum. After having found this horizontal component of the cable tension and the reactions under the stiffening trusses, the stresses in all the members of the entire structure can be at once determined. It is obvious that the stiffening truss and the cables must deflect together. It is equally evident that the deeper the stiffening trusses are the more load will be required to deflect them to any given amount, and hence that the deeper they are the more load they will carry independently of the cable. It is desirable to throw as much of the duty of carrying loads upon the cables as possible. It therefore follows that the stiffening trusses should be made as shallow as the proper discharge of their stiffening duties will permit.

137. Stresses in Cables and Moments and Shears in Trusses.— The necessary limits of this discussion will not permit even the simplest analyses to be given. It is evident, however, that the greatest cable stresses will exist at the tops of the towers, and that if the horizontal component of cable tension be found by any proper method, the stress at any other point will be equal to that horizontal component multiplied by the secant of cable inclination to a horizontal line, it being supposed that the suspenders are found in a vertical plane.

If the stiffening trusses are jointed at the centre of the main span, as well as at the ends, the simple equations of statical equilibrium are sufficient in number to make all computations, for the reason that the centre pin-joint gives the additional condition that, whatever may be the amount or distribution of loading, the centre moment must be zero. If l is the length of main or centre span and p the moving load per linear foot of span, and if the stiffening trusses run from tower to tower, the following equations will give their greatest moments and shears both by the old and new theory of the stiffening truss.

	p = load per lin. ft., Old theory.	l = length of span in ft., New theory.	
Max. moment....	$M = 0.01856pl^2$	$M = 0.01652pl^2$	no centre
Max. shear.......	$S = \frac{1}{8}pl$	$S = \frac{1}{8}pl$	hinge.

With centre hinge $M = 0.01883pl^2$ and $S = \frac{1}{8}pl$

The details of the theory of stiffening trusses for suspension bridges have been well developed during the past few years and are fully exhibited in modern engineering literature. The long spans requiring stiffened suspension bridges are usually found over navigable streams, and hence those bridges must be placed at comparatively high elevations. This is illustrated by the clear height of 135 feet required under the East River suspension-bridge structures already completed and in progress. Furthermore, the heights of towers above the lowest points of the cables usually run from one eighth to one twelfth of the span. These features expose the entire structure to comparatively high wind pressures, which must be carefully provided against. This is done by the requisite lateral bracing between the stiffening trusses and by what is called the cradling of the cables. The latter expression simply means that the cables as they are built are swung out of a vertical plane and toward the axis of the structure, being held in that position by suitable details. The cables on opposite sides of the bridge are thus moved in toward each other so as to produce increased stability against lateral movement. Occasionally horizontal cables are stretched between the towers in parabolic curves in order to resist horizontal pressures, just as the main cables carry vertical loads. This matter of stability against lateral wind pressures requires and receives the same degree of careful consideration in design as that accorded to the effects of vertical loading. The same general observation applies also to the design of the towers.

138. Thermal Stresses and Moments in Stiffened Suspension Bridges.—All material used in engineering structures expands and contracts with rising and falling temperatures to such an extent that the resulting motions must be provided for in structures of considerable magnitude. In ordinary truss-bridges one end is supported upon rollers, so that as the span changes its length the truss ends move the required amount upon the rollers. In the case of stiffened suspension bridges, however, the ends of the cables at the anchorages are rigidly fixed, so that any adjustment required by change of temperature must be consistent with the change of length of cable between the anchorages. The backstays, which are those portions of the cables extending from the anchorages to the tops of the towers, expand and contract precisely as do the portions of the cable between the tops of the towers. As the cables lengthen, therefore, the sag or rise at the centre of the main span will be due to the change in the entire length of cable from anchorage to anchorage. In order to meet this condition it is usual to support the cables at the tops of the towers on seats called saddles which rest upon rollers, so as to afford any motion that may be required. Designs have been made in which the cables are fixed to the tops of steel towers. In such cases changes of temperature would subject the towers to considerable bending which would be provided for in the design.

The rise and fall at the centres of long spans of stiffened suspension bridges is considerable; indeed, for a variation of 120° Fahr. the centre of the New York and Brooklyn Bridge changes its elevation by 4.6 feet if the saddles are free to move, as intended. In the case of a stiffened suspension bridge designed to cross the North River at New York City with a main span of 3200 feet a variation of 120° Fahr. in temperature would produce a change of elevation of the centre of the span of 6.36 feet. Such thermal motions in the structure obviously will produce stresses of considerable magnitude in various parts of the stiffening trusses, all of which are invariably recognized and provided for in good design.

139. Formation of the Cables.—At the present time suspension-bridge cables are made by grouping together in one cylindrical mass a large number of so-called strands or individual small cables, each composed of a large number of parallel wires about one sixth of an inch in diameter. The four cables of the New York and Brooklyn Bridge are each composed of 19 strands, each of the latter containing 332 parallel wires, making a total of 6308 wires, the cables themselves being $15\frac{1}{2}$ inches in diameter. The wire is No. 7 gauge, i.e., 0.18 inch in diameter. In the new East River Bridge each of the four cables is $18\frac{1}{4}$ inches in diameter and contains 37 strands, each strand being composed of 208 wires all laid parallel to each other, or a total of 7696 wires. The size of the wire is No. 6 (Roebling) gauge, i.e., 0.192 inch in diameter. These strands are formed by laying wire by wire, each in its proper place. The strands are then bound together into a single cable, around which is tightly wound a sheathing or casing of smaller wire, 0.134 inch in diameter for the New York and Brooklyn Bridge. The tightness of this binding wire insures the unity of the whole cable, each wire having been placed in its original position so as to take a tension equal to that of each of the other wires. The suspension-rods are usually of wire cables and are attached by suitable details to the lower chords of the stiffening truss, also by specially designed clamps to the cable. The stiffening trusses are usually built with all riveted joints, so as to secure the greatest possible stiffness from end to end. The stiffened suspension bridge has been shown by experience, as well as by theory, to be well adapted to carry railroad traffic over long spans.

140. Economical Limits of Spans.—In the past, suspension bridges have, in a number of cases, been built for comparatively short spans, but it is well recognized among engineers that their economical use must be found for spans of comparatively great length. While definite lower limits cannot now be assigned to such spans, it is probable that with present materials of construction and with available shop and mill capacities the ordinary truss-bridge may be economically used up to spans approximately 700 to 800 feet, and that above that limit the cantilever system is economically applicable to lengths of span not yet determined but probably between 1600 and 2000 feet. The special field of economical employment of the long-span stiffened suspension bridge will be found at the upper limit of the cantilever system. So far as present investigations indicate, the stiffened suspension type of structure may be employed to advantage from about 1800 feet up to the maximum practicable length of span not yet assignable, but perhaps in the vicinity of 4000 feet. Obviously such limits are approximate only and may be pushed upward by further improvements in the production of material and in the enlargement of both shop and mill capacity.